LNCT GROUP OF COLLEGES

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Department: EX

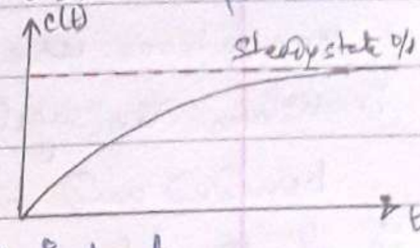
Subject: Control System (EX-405)

Unit: III

Topic: Stability Analysis

Stability of Control Systems –

– The stability of linear closed loop system can be determined from the locations of closed loop poles in the s-plane.

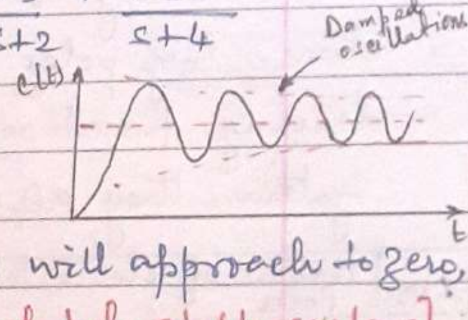


E.g. – Consider a C.L.T.F. $\frac{C(s)}{R(s)} = \frac{10}{(s+2)(s+4)}$

Let $R(s) = 1/s$, thus obtaining response for unit step input

$$C(s) = \frac{10}{s(s+2)(s+4)} = \frac{A}{s} + \frac{B}{s+2} + \frac{C}{(s+4)}$$

$$C(s) = 10\left[\frac{1/8}{s} - \frac{1/4}{s+2} + \frac{1/8}{s+4}\right] = \frac{1.25}{s} - \frac{2.5}{s+2} + \frac{1.25}{s+4}$$



$$c(t) = \underbrace{1.25}_{C_{ss}} - \underbrace{2.5e^{-2t} + 1.25\,e^{-4t}}_{c_t(t)\text{ - Transient Part}}$$

– Now when $t \to \infty$, both the exponential terms will approach to zero, and output will be steady state output. [Absolutely stable systems]

– The transient terms are exponential terms with –ve index because closed loop poles are located in left half of s-plane.

* If closed loop poles are located in left half of s-plane, exponential indices in output are negative, hence the exponential transient terms will vanish when $t \to \infty$

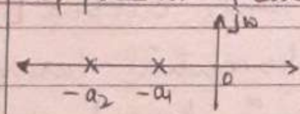
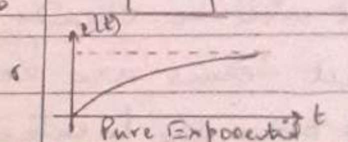
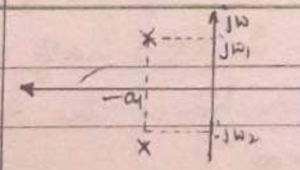
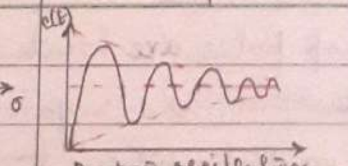
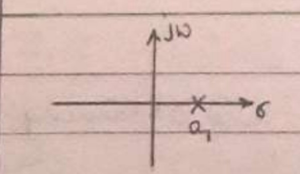
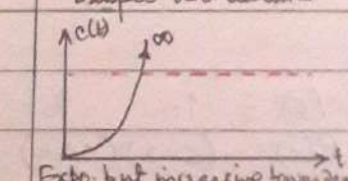
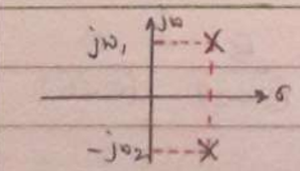
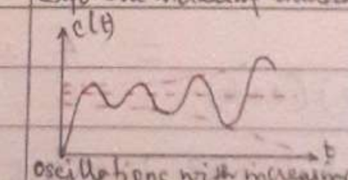
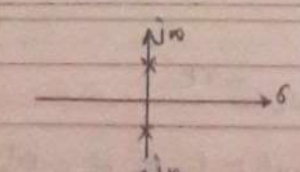
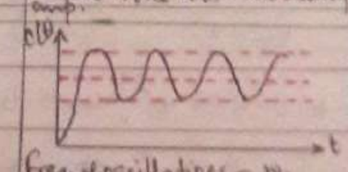
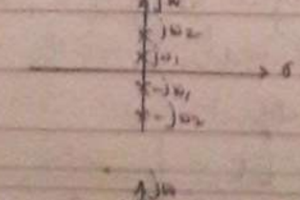
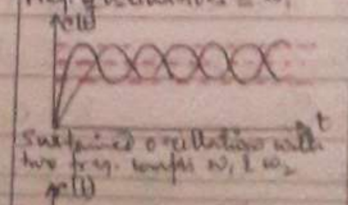
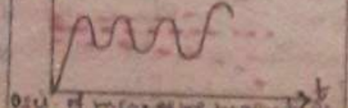
Definition of BIBO stability –

A linear time invariant system is said to be stable if following conditions are satisfied –

i) When the system is excited by a bounded input, output is also bounded and controllable.

ii) In the absence of input, output must tend to zero irrespective of the initial conditions

* The stability or instability is a property of the system itself i.e. closed loop poles of the system and doesn't depend on input or driving function. The poles of input do not affect stability of system, they affect only steady state output.

S.No.	Nature of closed loop Poles	Locations of closed loop poles in s-plane	Step response	Stability Condition
1.	Real, negative i.e. in LHS of s-plane			Absolutely stable
2.	Complex conjugate with -ve real part i.e. in L.H.S. of s-Plane.			Absolutely stable
3.	Real +ve, i.e. in RHS of s-Plane (Any one pole in RHS irresp. of total poles in LHS)			Unstable
4	Complex Conjugate with +ve real Part in RHS of s-plane			Unstable
5.	Non-repeated pair on imaginary axis without any pole in RHS of s-Plane			Marginally or critically stable
				Marginally or critically stable
6.	Repeated pair on imaginary axis without any pole in RHS of s-Plane			Unstable

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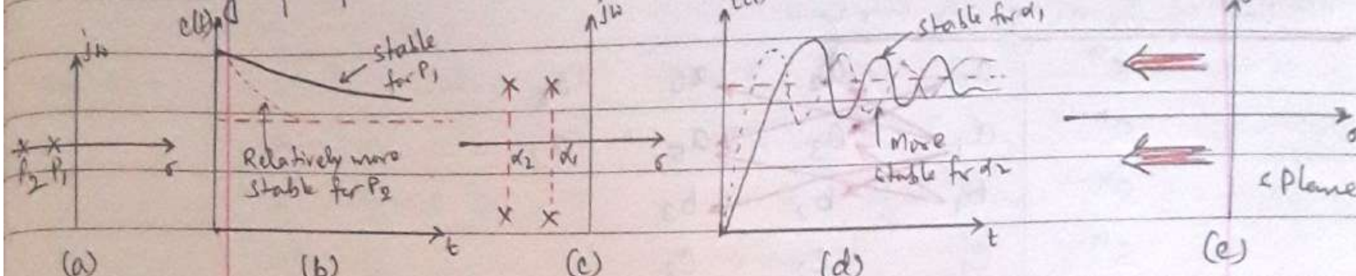
Relative Stability-

- The system is said to be relatively more stable or unstable on the basis of settling time.
- System is said to be relatively more stable if the settling time for that system is less than that of the other system.
- The settling time of the root or pair of complex conjugate roots is inversely proportional to the real part of the roots.



- So for roots located near the jω axis, settling time will be large. As roots or pair of complex conjugate roots move away from jω-axis i.e. towards left half of s-plane, settling time becomes lesser or smaller and system becomes more & more stable.

Routh-Hurwitz Criterion-

- This represents a method of determining the location of poles of a characteristic eq^n w.r.t. the left half and right half of the s-plane without actually solving the eq^n.

The T.f. of any linear C.L. system can be represented as-

$$\frac{C(s)}{R(s)} = \frac{b_0 s^m + b_1 s^{m-1} + \dots + b_m}{a_0 s^n + a_1 s^{n-1} + \dots + a_n} = \frac{B(s)}{F(s)}$$ where 'a' and 'b' are constants.

- To find closed loop poles denominator is equated to zero. This eq^n is called characteristic eq^n of the system.

i.e. $a_0 s^n + a_1 s^{n-1} + \dots + a_n = 0$

- Roots are the closed loop poles which decide the stability of the system.

Necessary Conditions-

1) All the coefficients of the polynomial have the same sign.
2) None of the coefficients vanish i.e. all powers of 's' must be present in decending order from 'n' to zero.

Routh's Stability Criterion –

- It is also called Routh's Array method or Routh Hurwitz's method.
- Routh suggested a method of tabulating the coefficients of characteristic eq^n in a particular way. Tabulation of coefficients gives an array called Routh's Array.
- Consider the general characteristic eq^n as –

$$F(s) = a_0 s^n + a_1 s^{n-1} + a_2 s^{n-2} + \cdots + a_n = 0$$

Method of forming an array –

$$\begin{array}{c|cccc}
s^n & a_0 & a_2 & a_4 & a_6 \ \cdots \\
s^{n-1} & a_1 & a_3 & a_5 & a_7 \ \cdots \\
s^{n-2} & b_1 & b_2 & b_3 & \\
s^{n-3} & c_1 & c_2 & c_3 & \\
\vdots & \vdots & \vdots & \vdots & \\
s^0 & a_n & & &
\end{array}$$

- Coefficients for first two rows are written directly from charact eq^n
- From these two rows, next rows can be obtained as follows

$$b_1 = \frac{a_1 a_2 - a_0 a_3}{a_1}, \quad b_2 = \frac{a_1 a_4 - a_0 a_5}{a_1}, \quad b_3 = \frac{a_1 a_6 - a_0 a_7}{a_1}$$

- From 2nd and 3rd row, 4th row can be obtained as –

$$c_1 = \frac{b_1 a_3 - a_1 b_2}{b_1}, \quad c_2 = \frac{b_1 a_5 - a_1 b_3}{b_1}$$

- This process continues till coefficients of s^0 are obtained which will be a_n.

Necessary Conditions –

- The necessary and sufficient condition for system to be stable is "All the terms in the first column of Routh's array must have same sign. There should not be any sign change in the 1st column of Routh's array.
- If there are any sign changes then,

a) System is unstable

b) The no. of sign changes equals the no. of roots lying in the RHS of the s-plane.

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Q1. Examine the stability of the following eq^n s using Routh's Method:

a) $s^3+6s^2+11s+6=0$ and b) $s^3+4s^2+s+16=0$

Sol^n a) –

s^3	1	11	0
s^2	6	6	0
s^1	10	0	
s^0	6		

* As there are no sign changes in the first column, hence the given system is stable.

Sol^n b) –

s^3	1	1	0
s^2	4	16	0
s^1	$\frac{4-16}{4}=-3$	0	
s^0	16		

* There are two sign changes hence the system is unstable. No. of poles on RHS of s-plane = 2 (No. of sign changes)

Special Cases of Routh's Criterion –

Case I – First element of any of the Rows of Routh array is zero and the same remaining row contains at least one non-zero element.

Effect – The terms in the new row becomes infinite and Routh's test fails.

eg:– $s^5+2s^4+3s^3+6s^2+2s+1=0$

s^5	1	3	2
s^4	2	6	1
s^3	$\boxed{0}$	1.5	0
s^2	∞	...	...
s^1			
s^0			

How to overcome – Substitute a small +ve number 'ε' in place of a zero occured as a first element in a row. Complete the array with this no. 'ε'. Then examine the change in sign by taking $\lim_{\epsilon \to 0}$.

s^5	1	3	2
s^4	2	6	1
s^3	ϵ	1.5	0
s^2	$\frac{6\epsilon-3}{\epsilon}$	1	0
s^1	$\frac{1.5(6\epsilon-3)/\epsilon-\epsilon}{(6\epsilon-3)/\epsilon}$	0	0
s^0	1		

* $\lim_{\epsilon\to 0} \frac{6\epsilon-3}{\epsilon} = \lim_{\epsilon\to 0} 6-\frac{3}{\epsilon} = 6-\infty = -\infty$

$\lim_{\epsilon\to 0} \frac{9\epsilon-4.5-\epsilon^2}{6\epsilon-3} = \frac{\cancel{-}4.5}{\cancel{-}3} = 1.5$

Routh array is –

s^5	1	3	2
s^4	2	6	1
s^3	ϵ	1.5	0
s^2	$-\infty$	1	0
s^1	$+1.5$	0	0
s^0	1	0	0

* As there are two sign changes hence the given system is unstable.

Case II – All the elements of a row in a Routh's array are zero.

Effect – The terms of the next row can't be determined and the Routh's test fails.

Eg:-

s^5	a	b	c
s^4	d	e	f
s^3	$\boxed{0 \quad 0 \quad 0}$		

→ Row of zeros.

How to overcome–

i) Form an equation by using the coefficients of a row which is just above the row of zeros. Such an eq^n is called an Auxiliary Equation denoted as A(s) –

$$A(s) = ds^4 + es^2 + f$$

* The coefficients of any row are corresponding to alternate powers of 's' starting from the power indicated against it

ii) Take the derivative of the auxiliary eq^n w.r.t. 's'

i.e. $\frac{dA(s)}{ds} = 4ds^3 + 2es$

iii) Replace row of zeros by the coefficient of $\frac{dA(s)}{ds}$

s^5	a	b	c
s^4	d	e	f
s^3	4d	2e	0

iv) Check the total changes in sign.

* Auxiliary eq^n is always the part of the original characteristic eq^n. This means the roots of Aux. Eq^n. are some of the roots of original characteristic eq^n. Roots of the Aux. Eq^n. are the most dominant roots of the original characteristic eq^n, from stability point of view.

Marginal K and frequency of sustained oscillations-

- Marginal value of 'K' is that value of 'K' for which system becomes marginally stable.

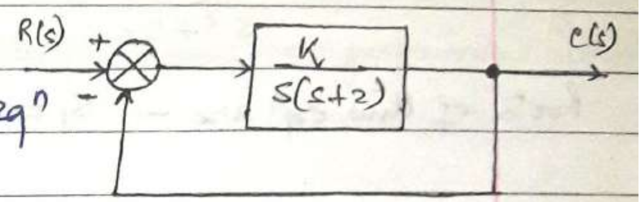
- For marginal stability, there must be a row of zeros occurring in Routh's array. So, value of 'K' which makes any row of Routh array as row of zeros is called marginal value of K.

- To obtain the frequency of oscillations, solve the auxiliary equation $A(s)=0$ for $K = K_{mar}$. The magnitude of imaginary roots of $A(s)=0$ obtained for marginal value of $K (K_{mar})$ indicates the frequency of sustained oscillations, which system will produce.

ROOT LOCUS TECHNIQUE

- The locus of the roots of the characteristic eq^n when gain is varied from zero to infinity is called Root Locus.

- Consider a unity feedback system as shown in fig. The characteristic eq^n is $1 + G(s)\,H(s) = 0$



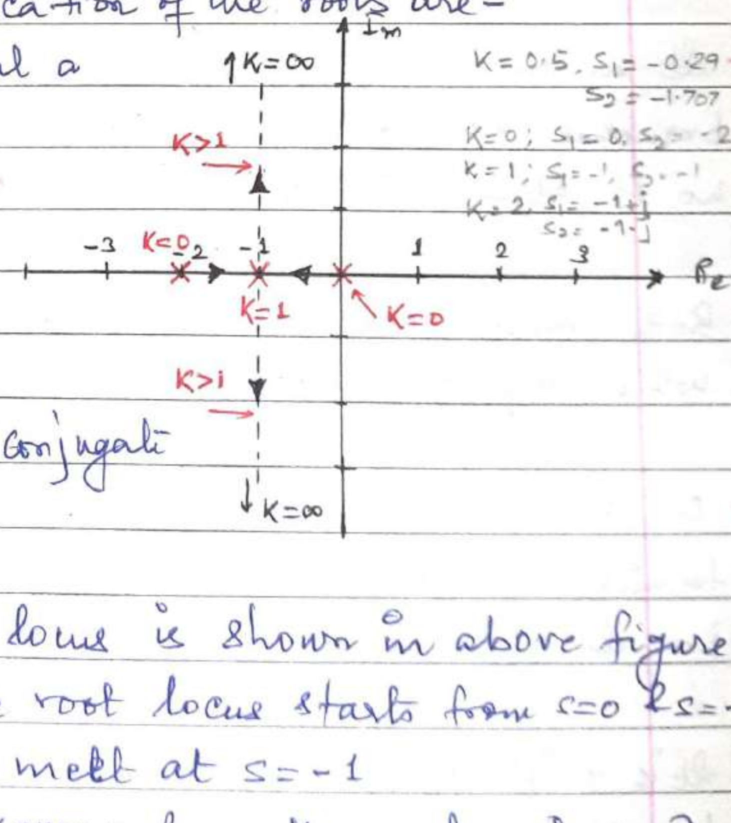
where $G(s) = \dfrac{K}{s(s+2)}$ and $H(s) = 1$

$$\therefore\ 1 + \frac{K}{s(s+2)} \cdot 1 = 0 \quad \text{or} \quad s^2 + 2s + K = 0 \quad \text{— (1)}$$

Roots of eq^n(1) are – $s_1 = -1 + \sqrt{1-K}$, $s_2 = -1 - \sqrt{1-K}$

- On varying the value of 'K', the two roots give the locii in s-plane. For various values of 'K', the location of the roots are –

1) when $0 < K < 1$, the roots are real and distinct
2) When $K = 0$, the roots are $s_1 = 0$ and $s_2 = -2$
3) When $K = 1$, both roots are real and equal.
4) when $K > 1$, roots are complex conjugate with real part $= -1$



- When K is varying the root locus is shown in above figure.
1) When $K = 0$, two branches of the root locus starts from $s=0$ & $s=-2$
2) When $K = 1$, both the roots meet at $s = -1$
3) When $K > 1$, the roots breakaway from the real axis and become complex conjugate having –ve real part equal to -1.

- No. of branches is equal to the no. of open loop poles.

Consider $G(s)H(s) = \dfrac{K(s+1)}{s(s+5)}$ and obtain the nature of root locus.

- Characteristic eq^n is – $1 + G(s)H(s) = 0$

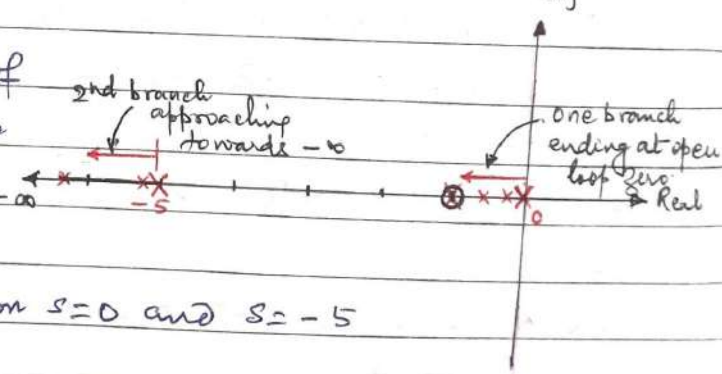
  i.e. $1 + \dfrac{K(s+1)}{s^2+5s} = 0$ or $s^2 + (5+K)s + K = 0$

Roots of this eq^n are – $s_1 = -\dfrac{(K+5)}{2} + \dfrac{\sqrt{K^2+6K+25}}{2}$ & $s_2 = -\dfrac{(K+5)}{2} - \dfrac{\sqrt{K^2+6K+25}}{2}$

- Effect of variation of K –

K	$s_1 = -\dfrac{(K+5)}{2} + \dfrac{\sqrt{K^2+6K+25}}{2}$	$s_2 = -\dfrac{(K+5)}{2} - \dfrac{\sqrt{K^2+6K+25}}{2}$
0	0	−5
1	−0.1715	−5.828
5	−0.527	−9.472
⋮		
∞	−1	−∞

- It can be observed that no. of branches are again two i.e. the no. of open loop poles.



- Both branches are starting from $s=0$ and $s=-5$ which are open loop poles.

- One of the important observations is that, one of the branches terminates at $s=-1$, which is an open loop zero, while the other branch is terminating at infinity.

- It is difficult to plot the root locus for higher order systems by substituting different values of 'K' in the roots of characteristic equation as used above.

- To simplify the construction of root locus for higher order systems certain rules are developed.

RULES FOR CONSTRUCTING ROOT LOCUS-

RULE 1:- The root locus is always symmetrical about the real axis.
The roots of the characteristic eqn are either real or complex conjugates or combination of both. Therefore, their locus must be symmetrical about the real axis of the s-plane.

RULE 2:- Let $G(s) \cdot H(s)$ = open Loop transfer function of the system
and P = No. of open loop poles ; Z = No. of open loop zeros.
i) If $P > Z$ i.e. no. of poles is greater than zeros, then no. of branches equals the no. of open loop poles (i.e. $N = P$)
ii) If $Z > P$ then $N = Z$ [No. of branches equals the no. of O.L zeros]

- for case (i) - Branches will start from each of the location of open loop pole. Out of 'P' no. of branches, 'Z' no. of branches will terminate at the locations of open loop zeros and the remaining 'P-Z' branches will approach to infinity.

- for case ii) - Branches will terminate at each of the finite location of open loop zero. But out of 'Z' no. of branches, 'P' no. of branches will start from each of the finite open loop pole locations while remaining 'Z-P' branches will originate from infinity & will approach to finite zeros.

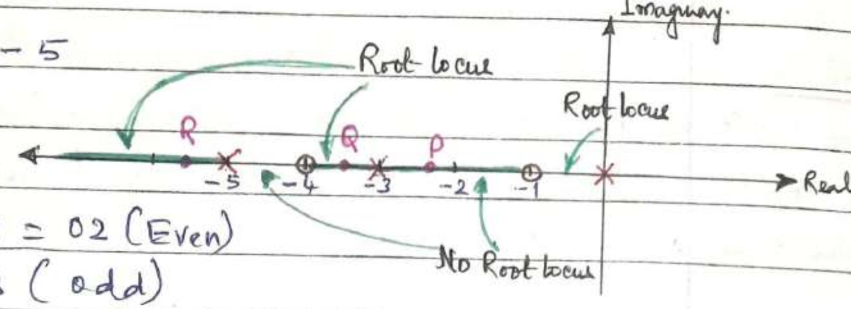
- When $P = Z$, the no. of branches are $N = P = Z$. A separate branch will start from each of the O.L. pole while will terminate at available each O.L. zero. No branch will start or terminate at infinity for $P = Z$.

RULE 3:- Root Loci on the Real Axis
Any point on the real axis is a part of the root locus if and only if the no. of poles and zeros sum is odd to its right.
- Complex poles or zeros are not considered while applying this rule.

Eg:- $G(s)H(s) = \frac{K(s+1)(s+4)}{s(s+3)(s+5)}$, find the sections of real axis where the RL lies.

Sol^n - Poles - $s = 0, -3, -5$

Zeros - $s = -1, -4$



For pt. P - Sum of poles & zeros to its right $= 1+1 = 02$ (Even)

For Q - $2(P) + 1(Z) = 03$ (odd)

For R - $3(P) + 2(Z) = 05$ (odd)

RULE 4 :- Asymptotes -

The branches of root locus tend to infinity along a set of straight line called asymptotes. The total no. of asymptotes $= P - Z$

- Angle made by the asymptotes with real axis is given by-

$$\theta = \frac{(2K+1)\,180°}{P-Z}$$

where $K = 0, 1, 2 \ldots (P-Z-1)$

RULE 5 :- Centroid of Asymptotes

The point of intersection of asymptotes with real axis is called centroid of asymptotes (σ) and is given by-

$$\sigma = \frac{\text{Sum of poles} - \text{Sum of Zeros}}{P-Z}$$

or $$\sigma = \frac{\sum \text{Real parts of poles of } G(s)H(s) - \sum \text{Real parts of zeros of } G(s)H(s)}{P-Z}$$

RULE 6 :- Breakaway Point -

It is a point on the root locus where multiple roots of the characteristic equation occurs, for a particular value of K.

- Obtain the characteristic equation $1 + G(s)\cdot H(s) = 0$ of the system.
- Obtain the expression of 'K' in terms of 's' i.e. $K = f(s)$
- Differentiate above eq^n w.r.t. 's' and equate it to zero.

  i.e. $\frac{dK}{ds} = 0$
- Roots of the eq^n $\frac{dK}{ds} = 0$ gives the breakaway point.

- If the value of K is positive, then that breakaway point is valid for the root locus. The breakaway points for which values of 'K' are -ve, those points are invalid.

RULE 7:- Intersection of root locus with imaginary axis-

Steps to be followed-

- Consider the characteristic eqⁿ obtained in RULE 6.
- Construct Routh's Array in terms of 'K'.
- Determine Kmarginal i.e. the value of K for which one of the rows of Routh's array becomes zero, except the row s^0.
- Construct auxiliary eqⁿ $A(s)=0$ by using coefficients of a row just above the row of zeros.
- Roots of auxiliary eqⁿ $A(s)=0$ for $K=K_{mar}$ are nothing but the intersection points of the root locus with imaginary axis.

* If K_{mar} is +ve, root locus intersects with imaginary axis else doesn't intersect with imaginary axis and totally lies on the left half of s-plane.

RULE 8:- Angle of Departure at complex pole-

The angle at which a complex conjugate pole departs is known as angle of departure, denoted by ϕ_d.

where $\phi_d = 180° - \left[\sum \phi_P - \sum \phi_Z\right]$

$\sum \phi_P$ = Contributions by angles made by remaining O.L. poles at the pole where ϕ_d is to be calculated.

$\sum \phi_Z$ = Contributions by the angles made by the O.L. zeros at the pole where ϕ_d is to be calculated.

RULE 9:- Angle of arrival at complex zero-

$$\phi_a = 180° \left[\sum \phi_Z - \sum \phi_P\right]$$

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Q1. Draw the approximate root locus diagram for the closed loop system whose open loop transfer function is given by -

$$G(s).H(s) = \frac{K}{s(s+5)(s+10)}$$. Comment on the stability.

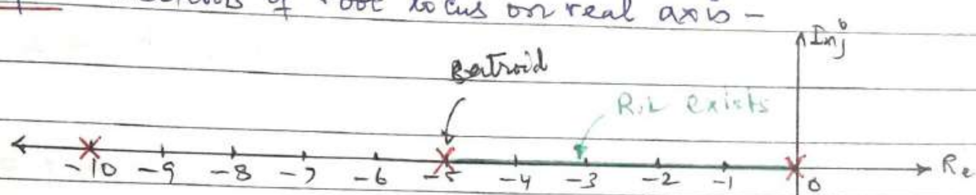
Solⁿ - Step 1 - $P=3$, $Z=0$, $N=P=3$ branches.

$P-Z = 3$ branches approaching ∞

Starting points $= 0, -5, -10$

Terminating points $= \infty, \infty, \infty$

Step 2 - Sections of root locus on real axis -



On RHS of Pole at $s=-5$, $P+Z = 1$ (odd)

On RHS of Pole at $s=-10$, $P+Z = 1+1 = 2$ (Even)

Step 3 - Angle of Asymptotes

$$\theta = \frac{(2K+1)180^\circ}{P-Z} , \quad K = 0,1,2$$

$$\theta_1 = \frac{180^\circ}{3} = 60^\circ, \quad \theta_2 = \frac{3\times180^\circ}{3} = 180^\circ, \quad \theta_3 = \frac{5\times180^\circ}{3} = 300^\circ$$

Step 4 - Centroid of Asymptotes -

$$\sigma = \frac{\Sigma \text{R.P. of poles} - \Sigma \text{R.P. of zeros}}{P-Z} = \frac{0-5-10-0}{3} = -5$$

Step 5 - Break away Point

$$1 + G(s)H(s) = 0 \quad \text{or} \quad 1 + \frac{K}{s(s+5)(s+10)} = 0$$

or $s^3 + 15s^2 + 50s + K = 0$ or $K = -s^3 - 15s^2 - 50s$ — (1)

$$\frac{dK}{ds} = -3s^2 - 30s - 50 = 0 \quad \text{or} \quad s^2 + 10s + 16.667 = 0$$

$$\therefore s = \frac{-10 \pm \sqrt{(10)^2 - 4\times16.67}}{2} = -2.113, -7.88$$

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Substituting $s = -2.113$ in eq^n(1), $K = 48.112$ (valid)
for $s = -7.88$, $K = -48.112$ (Invalid)

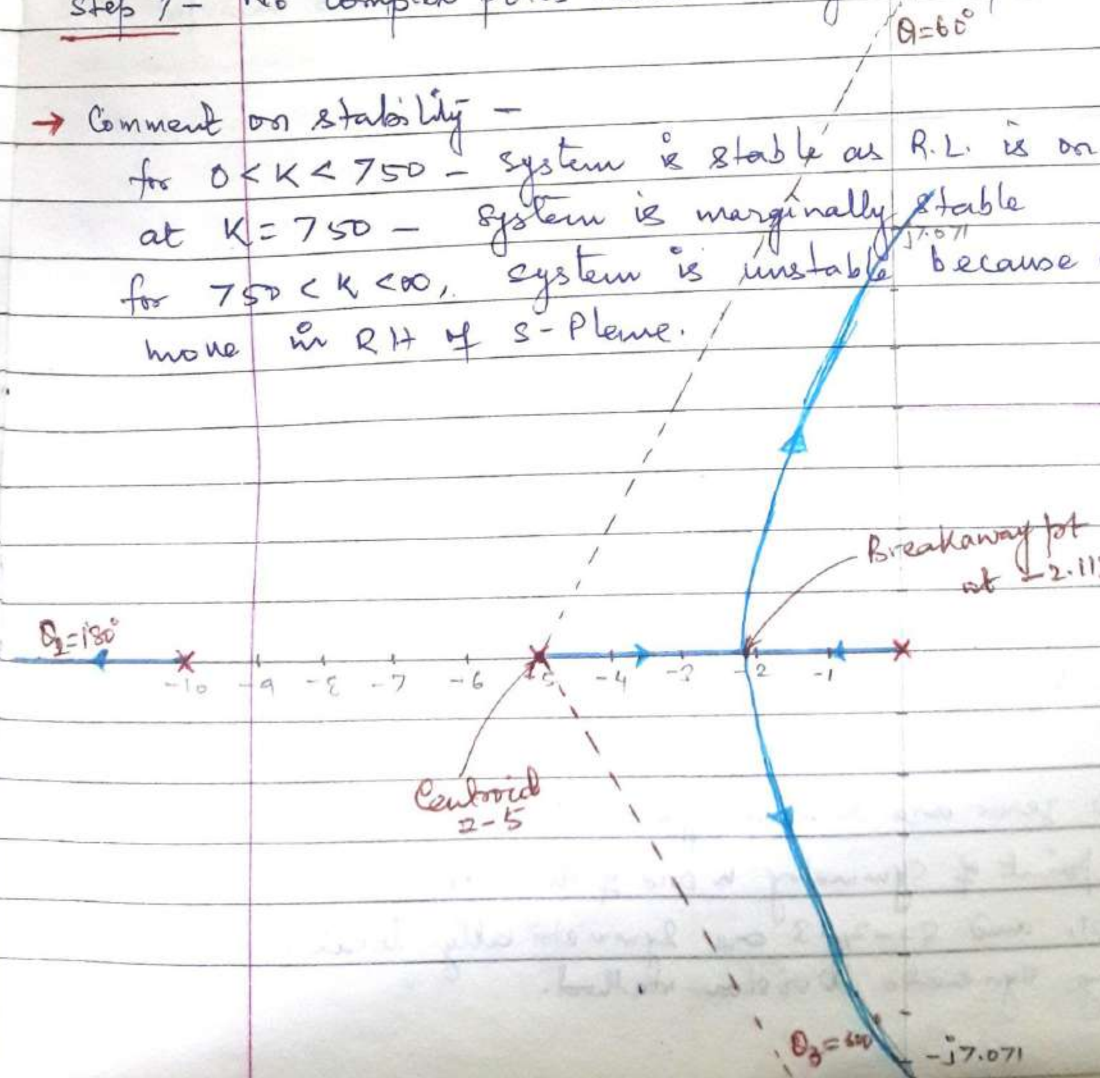
Step 6 – Intersection with imaginary axis
characteristic eq^n $s^3 + 15s^2 + 50s + K = 0$

s^3	1	50
s^2	15	K
s^1	$\frac{750-K}{15}$	0
s^0	K	

form row of s^1, $750 - K = 0$ or $K = 750$ $\therefore K_{max} = 750$
$A(s) = 15s^2 + K = 0$ or $15s^2 + 750 = 0$
or $s = \pm j\sqrt{50} = \pm j7.071$

Step 7 – No complex poles hence no angle of departure required.

→ Comment on stability –
for $0 < K < 750$ – system is stable as R.L. is on the L.H.S. of s-plane
at $K = 750$ – system is marginally stable
for $750 < K < \infty$, system is unstable because dominant poles move in RH of s-Plane.



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Q2. sketch the complete root locus of the system having O.L.T.F -

$$G(s)\,H(s) = \frac{K}{s(s+1)(s+2)(s+3)}$$

Sol^n Step 1 - $P=4$, $Z=0$, $N=4$ and all the branches will approach to ∞

starting points, $s = 0, -1, -2, -3$

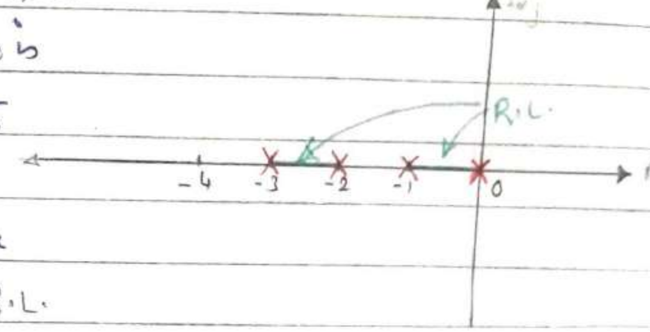
Step 2 - Sections of R.L. on real axis

Section b/w 0 and -1 → R.L. exists

section b/w -1 and -2 → No R.L

Section b/w -2 and -3 → R.L. exists

Section b/w -3 and above - No R.L.



Step 3 - Angle of Asymptotes -

$$\theta = \frac{(2K+1)\,180^\circ}{P-Z} \quad \text{where } K = 0, 1, 2, 3$$

for $K=1$, $\theta_1 = 45^\circ$; for $K=2$, $\theta_2 = 135^\circ$; for $K=3$, $\theta_3 = 225^\circ$ and for $K=3$, $\theta_4 = 315^\circ$

Step 4 - Centroid of Asymptotes -

$$\sigma = \frac{\sum \text{R.P. of poles} - \sum \text{R.P. of zeros}}{P-Z}$$

$$\sigma = \frac{0-1-2-3}{4} = \frac{-6}{4} = -1.5$$

Step 5 - Breakaway points -

characteristic Eq^n is $1 + G(s)H(s) = 0$

or $1 + \frac{K}{s(s+1)(s+2)(s+3)} = 0$

$s^4 + 6s^3 + 11s^2 + 6s + K = 0$ or $K = -s^4 - 6s^3 - 11s^2 - 6s$.

Now $\frac{dK}{ds} = -4s^3 - 18s^2 - 22s - 6 = 0$ or $4s^3 + 18s^2 + 22s + 6 = 0$

* If O.L poles and zeros are located symetrically about a point on the real axis then the point of symmetry is one of the roots of eq^n $\frac{dK}{ds} = 0$

Here roots $s = 0, -1$, and $s = -2, -3$ are symmetrically located about point $s = -1.5$. Using synthetic division method.

−1.5	4	18	22	6
		−6	−18	−6
	4	12	4	$\boxed{0}$

$\therefore$ $(s+1.5)(4s^2+12s+4) = 0$

Now $4s^2+12s+4=0$ or $s^2+3s+1=0$

or $s = \dfrac{-3 \pm \sqrt{9-4\times1}}{2} = \dfrac{-3\pm\sqrt{5}}{2} = -0.381, -2.618$

$\therefore$ Roots of $\dfrac{dK}{ds}=0$ are $s=-1.5, -0.381, -2.618$

* But no root locus exists b/w −1 and −2 hence −1.5 can't be a valid breakaway pt. And for $s=-0.381$ and -2.618 the value K is +ve, hence both are valid breakaway points.

Step 6 – Intersection with imaginary axis

characteristic eqn – $s^4+6s^3+11s^2+6s+K=0$

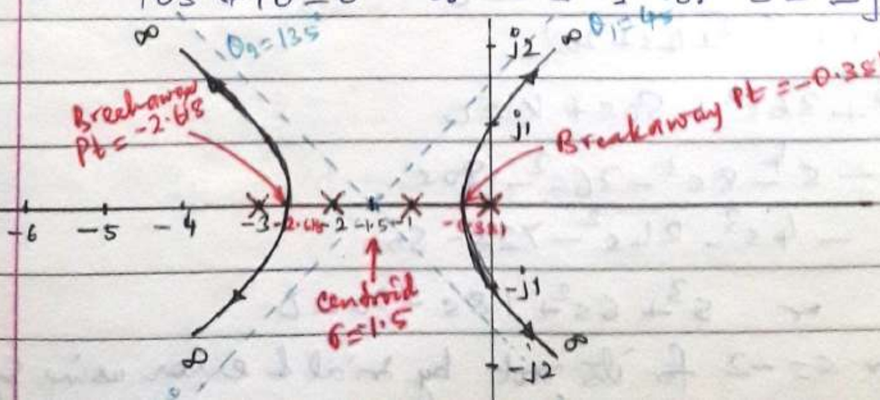
Routh Array –

s^4	1	11	K
s^3	6	6	0
s^2	10	K	0
s^1	$\frac{60-6K}{10}$	0	0
s^0	K		

$\therefore$ $\dfrac{60-6K}{10}=0$ $\therefore$ $K_{max}=10$

Auxiliary eqn is $A(s) = 10s^2+K=0$, so for $K=10$

$10s^2+10=0$ or $s^2=-1$ or $s=\pm j$



Assignment on Unit-III Stability Analysis

Q1. Determine the stability of the following system with characteristic equations given by:

a) $s^5 + s^4 + 2s^3 + 2s^2 + 3s + 15 = 0$

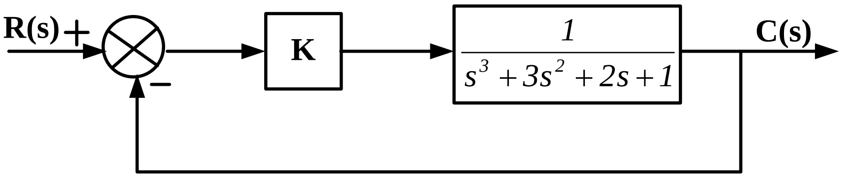
b) $s^6 + 2s^5 + 8s^4 + 12s^3 + 20s^2 + 16s + 16 = 0$

Q2. For the following given characteristic equations find the no. of roots with positive real part, zero real part and negative real part?

a) $s^6 + 4s^5 + 3s^4 - 16s^2 - 64s - 48 = 0$

b) $s^6 + 3s^5 + 4s^4 + 6s^3 + 5s^2 + 3s + 2 = 0$

Q3. A control system is shown below. Find the range of 'K' for which the system is stable?



Q4. Find the range of values of 'k' for the system to be stable whose characteristic equation is: $s^3 + 3ks^2 + (k+2)s + 4 = 0$

Q5. Sketch the root locus for the following systems with the given open loop transfer functions:

a) $G(s)H(s) = \dfrac{K(s+4)}{s(s^2 + 2s + 2)}$

b) $G(s)H(s) = \dfrac{K}{s(s+3)(s^2 + 2s + 2)}$

c) $G(s)H(s) = \dfrac{K}{s(s+3)(s+5)}$

d) $G(s)H(s) = \dfrac{K}{s(s^2 + 4s + 1)}$

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